NETWORK THEOREMS

- **Thevenin's Theorem**
- **Norton's Theorem**
- **Maximum Power Transfer Theorem**
- **Superposition Theorem**
- **Reciprocity Theorem**
- **Tellegen's Theorem**
- **Substitution Theorem**
- **Compensation Theorem**
- **Millman's Theorem**

INTRODUCTION:

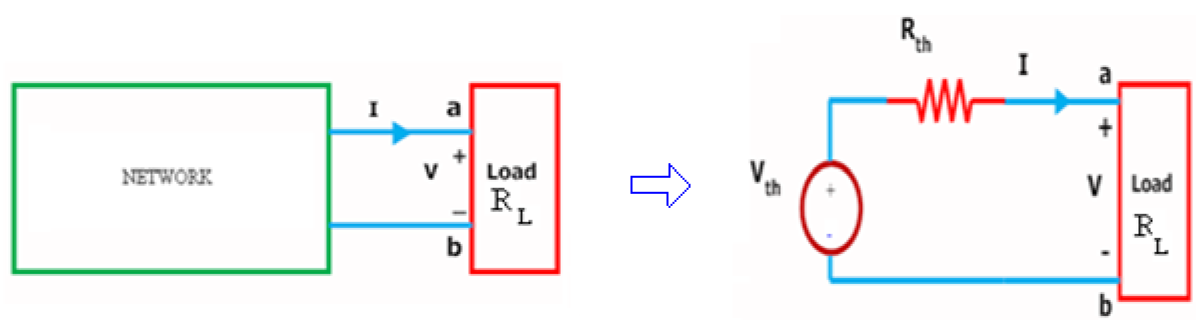
Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

Thevenin's Theorem and Norton's theorem (Introduction) :

Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the networks can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when it's value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source $\mathbf{V_{Th}}$ is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance $\mathbf{R_{Th}}$ while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.



(a) (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance $\mathbf{R_L}$ connected across the terminals **'a & b'** and figure (b) shows the **Thevenin equivalent circuit** with $\mathbf{V_{Th}}$ connected across $\mathbf{R_{Th}}$ & $\mathbf{R_L}$ **.**

Main steps to find out $\mathbf{V_{Th}}$ and $\mathbf{R_{Th}}$:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

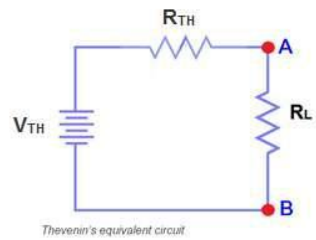
2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{Th} **.**

3. **Thevenin resistance** R_{Th} is found out by the method depending upon whether the network contains dependent sources or not.

    a. With dependent sources: $R_{Th} = V_{oc} / I_{sc}$

    b. Without dependent sources : R_{Th} **= Equivalent resistance looking into the concerned terminals** with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with V_{Th} in series with R_{Th} and the concerned branch resistance **(or)** load resistance across the load terminals(A&B) as shown in below fig.



Example: Find V_{TH}, R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?

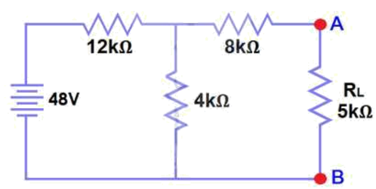


Fig.(a)

Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as **A & B** as shown in the fig. (1)

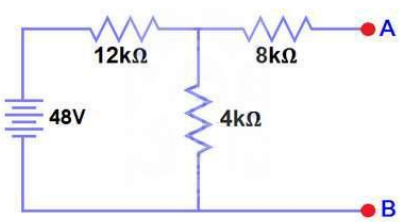
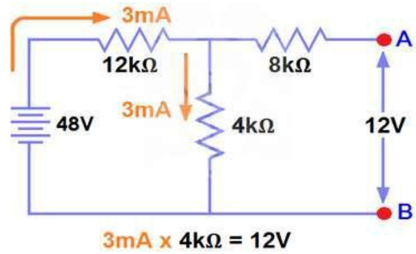


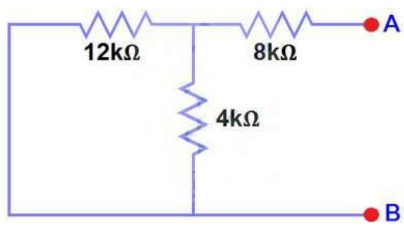
Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals. So, **V_{TH} = 12V**



Fig(2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)



Fig(3)

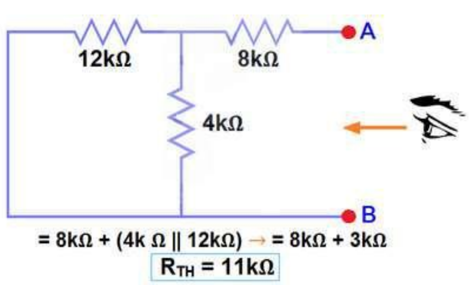
Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH})We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

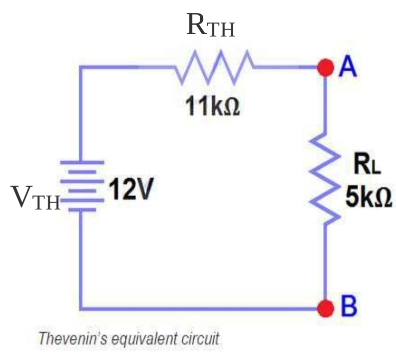
R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ



Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit



Fig(5)

Now apply Ohm's law and calculate the total load current from fig 5.

$I_L = V_{TH}/ (R_{TH} + R_L)= 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$

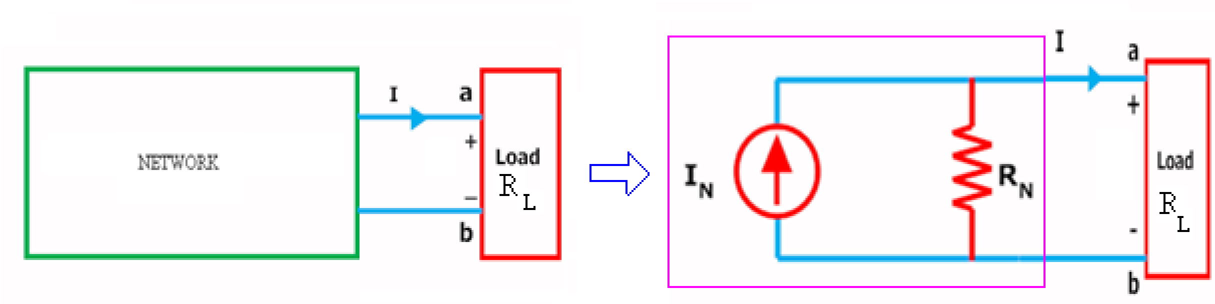
I_L= 0.75mA

And $V_L = I_L x\ R_L$= 0.75mA x 5kΩ

V_L= 3.75V

Norton's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



(a) (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance **R_L** connected across the terminals **'a & b'** and figure (b) shows the **Norton equivalent circuit** with **I_N** connected across **R_N** & **R_L .**

Main steps to find out I_N and R_N:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

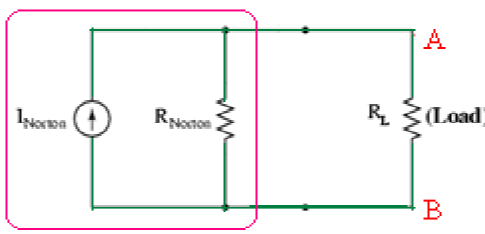
2. Open circuit voltage $\mathbf{V_{OC}}$ across these two terminals and $\mathbf{I_{SC}}$ through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit.

3. Next **Norton resistance** $\mathbf{R_N}$ is found out depending upon whether the network contains dependent sources or not.

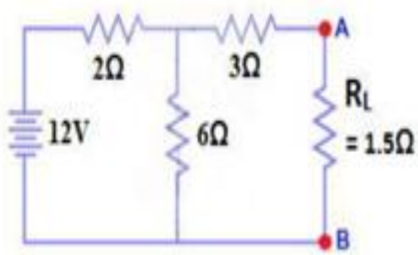
   a) With dependent sources: $\mathbf{R_N = V_{oc} / I_{sc}}$

   b) Without dependent sources : $\mathbf{R_N}$ = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with $\mathbf{I_N}$ in parallel with $\mathbf{R_N}$ and the concerned branch resistance across the load terminals(A&B) as shown in below fig

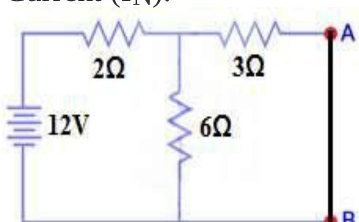


Example: Find the current through the resistance R_L (1.5 Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit.?



Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $\mathbf{I_N = I_{sc}}$ **,**$\mathbf{R_N = V_{oc} / I_{sc}}$**.** Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with)

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)]

R_T = 2Ω + 2Ω

R_T = 4Ω

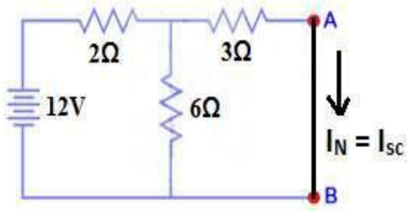
I_T = V / R_T

I_T = 12V / 4Ω= 3A..

Now we have to find I_{SC} = I_N… Apply CDR… (Current Divider Rule)…

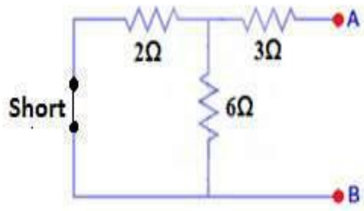
I_{SC} = I_N = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

I_{SC}= I_N = 2A.



Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)



Fig(4)

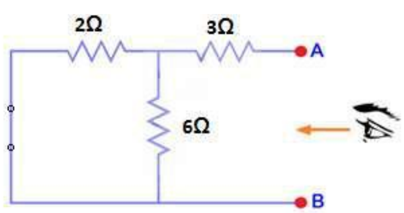
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

R_N = 3Ω + [(6Ω x 2Ω) / (6Ω + 2Ω)]

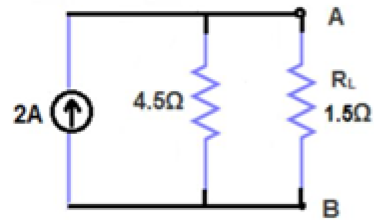
R_N = 3Ω + 1.5Ω

R_N = 4.5Ω



Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Fig(6)

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$I_L = I_N$ x $[R_N / (R_N + R_L)]$

$I_{L=}$ 2A x (4.5Ω /4.5Ω +1.5kΩ)

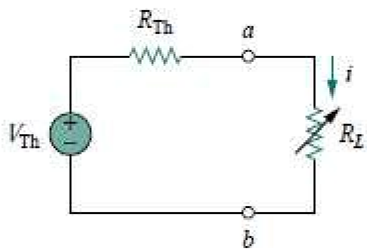
I_L = 1.5A **I_L = 1. 5A**

Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Maximum Power Transfer Theorem Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance , **R_L** must be equal to the source resistance i.e. Network's Thevenin equivalent resistance **R_{Th} . i.e. $R_L = R_{Th}$**

The load current **I** in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH}+R_L}$$

The power delivered by the circuit to the load:

$$P = I^2R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2}R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of **R_L** for maximum power transfer) and equating it to zero as :

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} - \frac{2V_{TH}^2}{(R_{TH}+R_L)^3}R_L = 0$$

Simplifying the above equation, we get:

$$(R_{TH} + R_L) - 2R_L = 0 \Longrightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by :

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

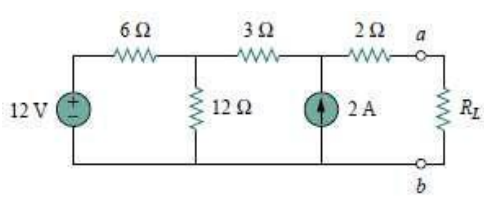
Under the condition of maximum power transfer, the efficiency $\boldsymbol{\eta}$ of the network is then given by:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

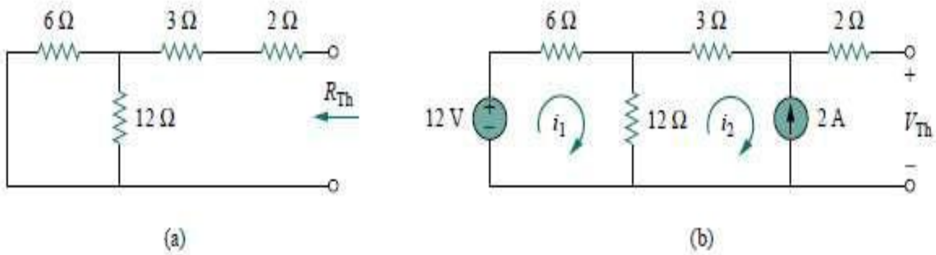
$$\boldsymbol{\eta} = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{(\frac{V_{TH}^2}{4R_L} + \frac{V_{TH}^2}{4R_L})} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50 %.

Example: Find the value of R_L for maximum power transfer in the circuit of Fig. Find the maximum power.?



Solution: We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. (a)



R_{Th}= 2 + 3 + (6 // 12)=5+($\frac{6\times12}{6+12}$)=5+4=9Ω

To get V_{Th}, we consider the circuit in Fig.(b).Applying mesh analysis,

$-12 + 18i_1 - 12i_2 = 0,$

$i_2 = -2$ A,

Solving for i1, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain,

$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$

V_{Th}= 22 V

For maximum power transfer, $R_L = R_{Th}$= 9Ω and the maximum power is,

$$P_{MAX} = \frac{V_{TH}^2}{4R_L} = \frac{22\times22}{4\times9} = 13.44\text{W}$$

Superposition Theorem:

The principle of superposition helps us to analyze a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Superposition Theorem Statement:

Any linear, bilateral two terminal network consisting of more than one sources, The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit)

Steps to Apply Super position Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?

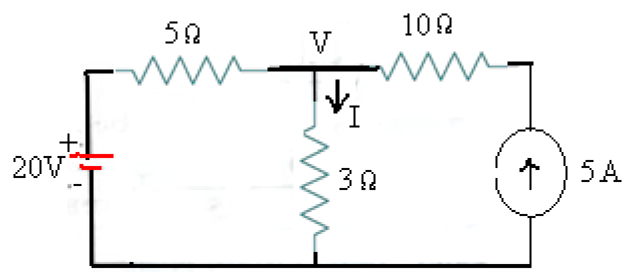


Fig.(a)

Solution: Applying the superposition theorem, the current $\mathbf{I_2}$ in the resistance of 3 Ω due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1 below] is given by :

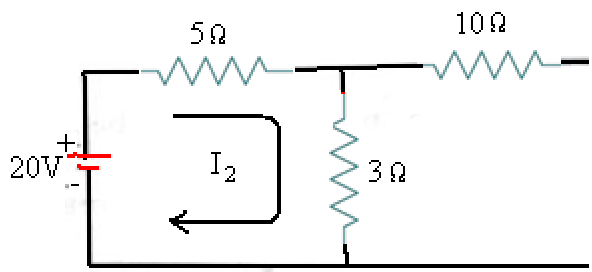


Fig1

$I_2 = 20/(5+3) = 2.5A$

Similarly the current I_5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by :

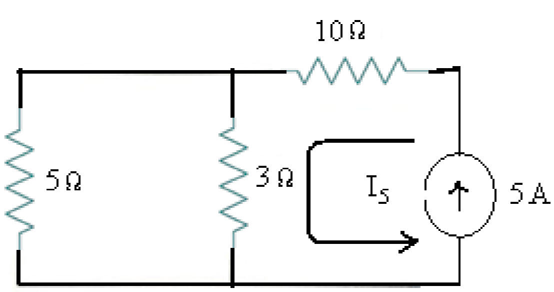


Fig.2

I_5= 5 x 5/(3+5) = 3.125 A

The total current passing through the resistance of 3Ω is then = I_2 + I_5= 2.5 + 3.125 = **5.625 A**

Let us verify the solution using the basic nodal analysis referring to the node marked with V in fig.(a).Then we get :

$$\frac{V-20}{5}+\frac{V}{3}=5$$

3V-60+5V=15× 5

8V-60=75

8V=135

V=16.875

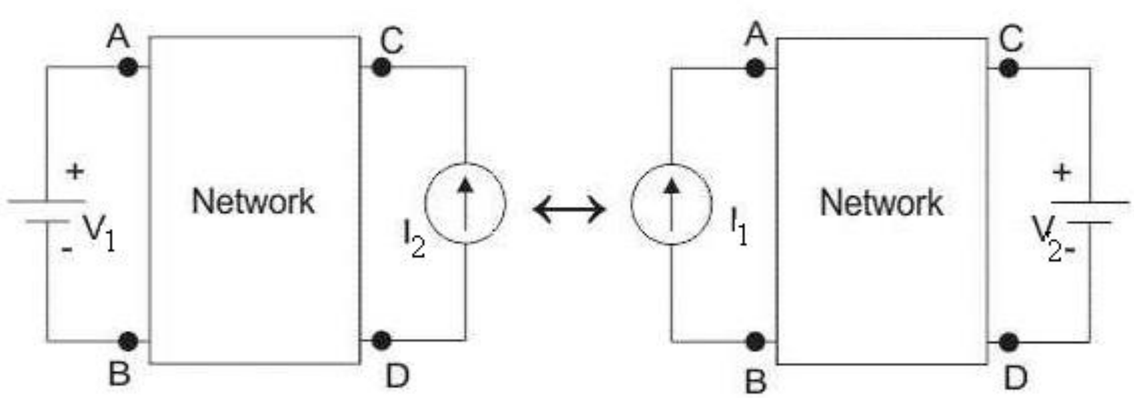
The current I passing through the resistance of 3Ω =V/3 = 16.875/3 = **5.625 A .**

Reciprocity theorem:

Under Basic Electrical Engineering In many electrical networks it is found that if the positions of voltage source and ammeter are interchanged, the reading of ammeter remains the same. Suppose a voltage source is connected to a passive network and an ammeter is connected to other part of the network to indicate the response. Now any one interchanges the positions of ammeter and voltage source that means he or she connects the voltage source at the part of the network where the ammeter was connected and connects ammeter to that part of the network where the voltage source was connected. The response of the ammeter means current through the ammeter would be the same in both the cases. This is where the property of reciprocity comes in the circuit. The particular circuit that has this reciprocal property, is called reciprocal circuit.

Reciprocity theorem Statement:

Any linear, bilateral two terminal network the ratio of excitation to response is constant even though the source is interchanged from input terminals to the output terminals.



$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

Steps For Solution Of a Network Utilizing Reciprocity Theorem:

1. The branches between which reciprocity is to be established to be selected first.
2. The current in the branch is obtained using conventional network analysis.
3. The voltage source is interchanged between the branches concerned.
4. The current in the branch where the voltage source was existing earlier is calculated.

It may observe that the currents obtained in 2 & 4 are identical to each other.

Example: Verify the reciprocity theorem for the network shown in the figure (1).?

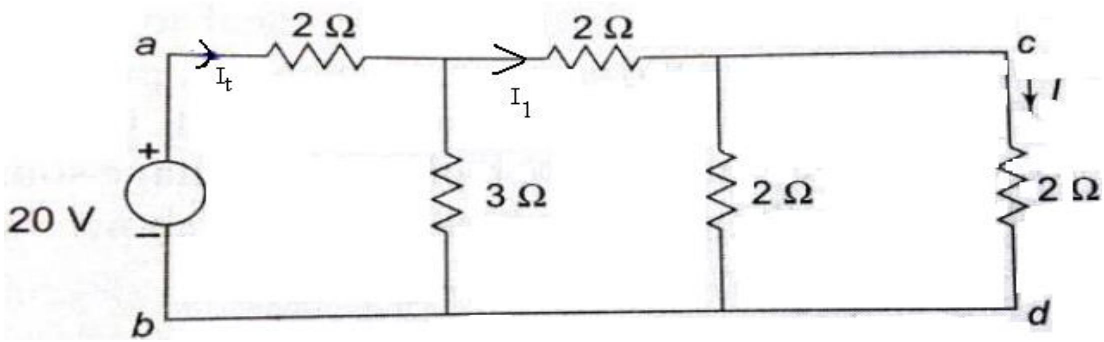


Fig.(1)

Solution: Total resistance in the circuit across the applied voltage of 20 V is

R_{TH}=2 + [3||(2 +(2||2))]

=2 +[3||3]

=3.5 Ω

The total current drawn by the circuit $\mathbf{I_T} = \frac{V}{R_{TH}}$= 20/3.5 = 5.71 A

The current **I** in the branch '**cd'** with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

$$I_1=5.71\times \frac{3}{3+3}=2.855A$$

The current I in the 'cd' branch is

$$I=2.855\times \frac{2}{2+2}=1.427A$$

Now the source voltage and the response are interchanged between branches **'ab'** and **'cd'** as shown in the figure (2) below

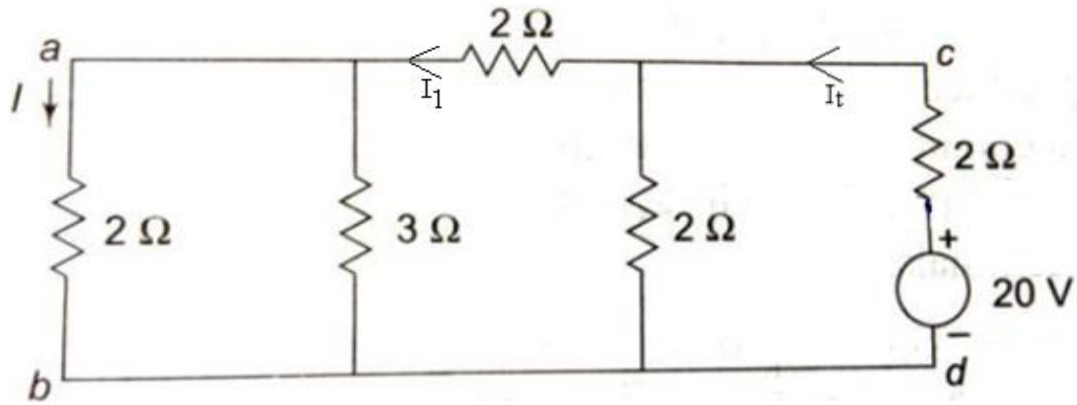


Fig.(2)

Total resistance in the circuit across the applied voltage of 20 V is

R_{TH}=2 + [2||(2 +(2||3))]

=2 +[2||3.2]

=3.23Ω

The total current drawn by the circuit $\mathbf{I_T} = \frac{V}{R_{TH}}$=20/3.23=6.19A

The current **I** in the branch **'ab'** with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

$$I_1 = 6.19 \times \frac{2}{3.2+2} = 2.38A$$

The current I in the 'ab' branch is

$$I = 2.38 \times \frac{3}{3+2} = 1.427A$$

The current in the branch **'ab' = 1.427 A** which is same as the current we got in branch **'cd'** when the voltage was given from branch **'ab' .** Thus the **reciprocity theorem** is verified.

Tellegen's Theorem:

This theorem is the one of the most general theorems in network analysis regardless to the type and nature, Tellegen's theorem is applicable to any network made up of lumped two terminal elements.

Tellegen's Theorem Statement:

In any linear, non-linear, passive, active, time variant or time invariant network the algebraic sum of power at any given instant is zero. Thus for K^{th} branch, this theorem states that

$$\sum_{K=1}^{n} v_K i_K = 0$$

Where, n=Being the number of branches,

V_K=Voltage drop in the branch,

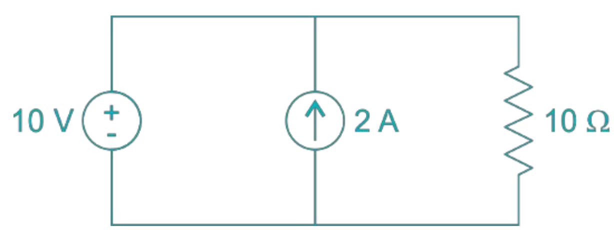
I_K=Current drop in the branch.

It also evident that the sum of power delivered to the network is equal to the sum of power absorbed by the network elements.

Steps For Solution Of a Network Utilizing Tellegen's Theorem:

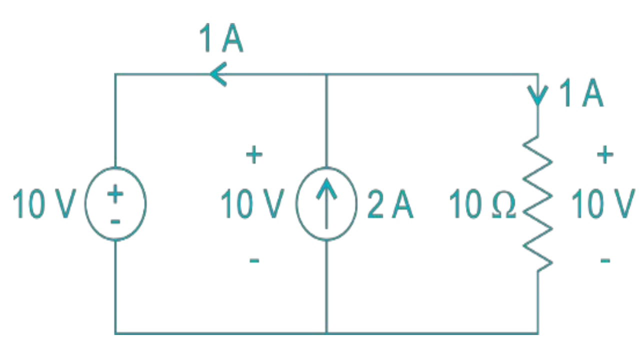
1. Find the branch voltage drops and corresponding branch currents using conventional analysis.
2. Summing all products of branch voltage and current.

Example: Verify the Tellegen's theorem for the given circuit.?



Solution: If current flows from + to – then treat it as power absorption.

If current flow from – to + then treat it as power delivering.



$\therefore$ P_{10V} = V. I = 10 × 1 = 10 watt ($P_{absorbed}$).

$P_{2A} = V.I = 10 \times 2 = 20$ watt ($P_{delivered}$)

$P_{10\Omega} = I^2.R = 1^2 \times 10 = 10$ watt ($P_{absorbed}$)

Pdelivered = Pabsorbed = 20 watt

Hence Tellegen's theorem is verified.

Substitution Theorem Statement:

The voltage across and the current through a Branch in a bilateral network is known, the branch can be replaced by any combination of elements in such a way that the same voltage will appear across and same current will pass through the chosen terminals. In other words for branch equivalence the terminal voltage and the current must be same.

This is illustrated with a simple circuit shown in the figure below.

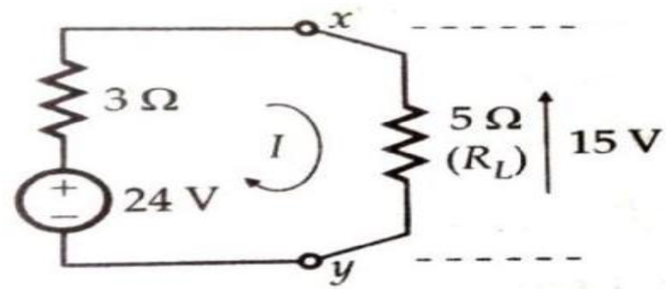


Figure: (a) A simple DC circuit to explain the substitution theorem.

In this circuit the load resistance $\mathbf{R_L}$ is the branch being considered for equivalence. The current **I** through the load resistance $\mathbf{R_L}$ **= 24/(3+5) = 3 A.**

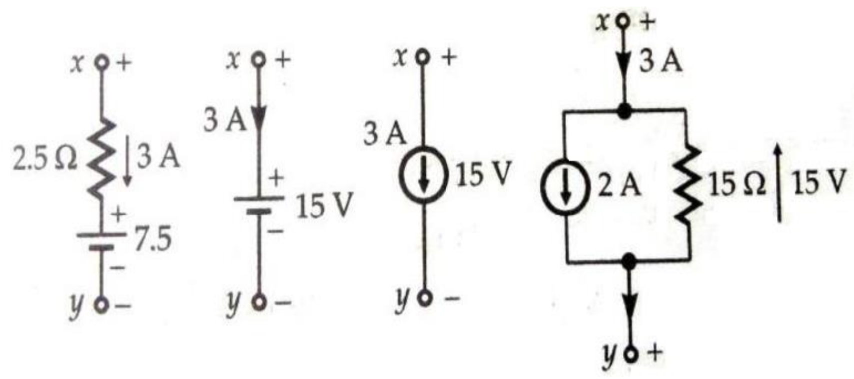


Figure: (b) Equivalent branches across terminals 'xy'

In figure (b) above several equivalents of branch **'xy'** are shown. It may be noted that in all the cases the terminal voltage across and the current through the equivalent branch are the same as that of the original branch $\mathbf{R_L}$**.** It may also be observed that a known potential difference and a current in a branch can be replaced by an ideal voltage source or an ideal current source respectively.

The limitation of this theorem is that it cannot be used to solve a network containing two or more sources that are not in series or parallel.

Example: Using substitution theorem, draw equivalent branches for the branch 'a-b' of the network of Fig.(a)?

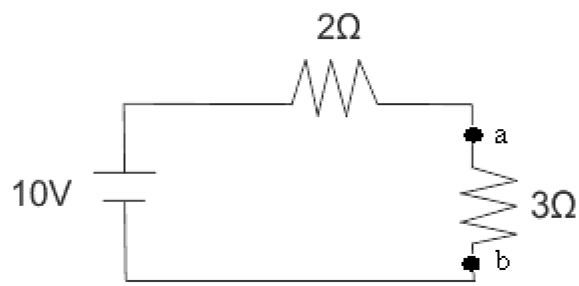


Fig.(a)

Solution: As per voltage division rule voltage across 3Ω and 2Ω resistance are

$$V_{3\Omega} = \frac{10 \times 3}{2+3} = 6V$$

$$V_{2\Omega} = \frac{10 \times 2}{2+3} = 4V$$

Current through the circuit is, $I = \frac{10}{2+3} = 2A$

If we replace the 3Ω resistance with a voltage source of 6 V as shown in fig (1), then

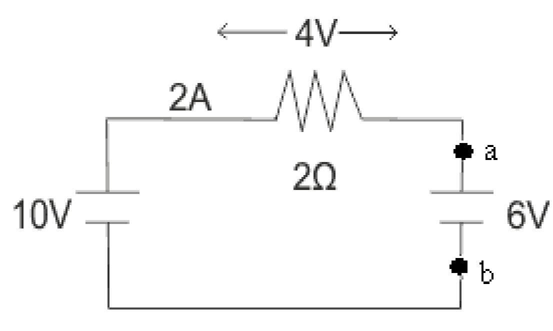


Fig.(1)

Then according to Ohm's Law the voltage across 2Ω resistance and current through the circuit is,

$$V_{2\Omega} = 10 - 6 = 4V$$

$$I = \frac{10-6}{2} = 2A$$

Alternately if we replace 3Ω resistance with a current source of 2A as shown in Fig(2),then

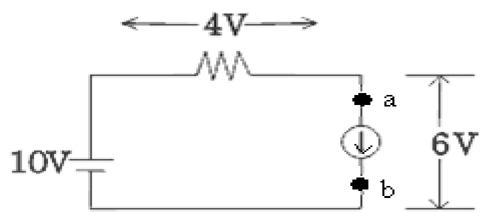


Fig.(2)

Voltage across 2Ω is $V_{2\Omega}$ = 10 – (3× 2) = 4 V and
Voltage across 2A current source is V_{2A} = 10 - 4 = 6 V.

The voltage across 2Ω resistance and current through the circuit is unaltered i.e. all initial condition of the circuit is intact.

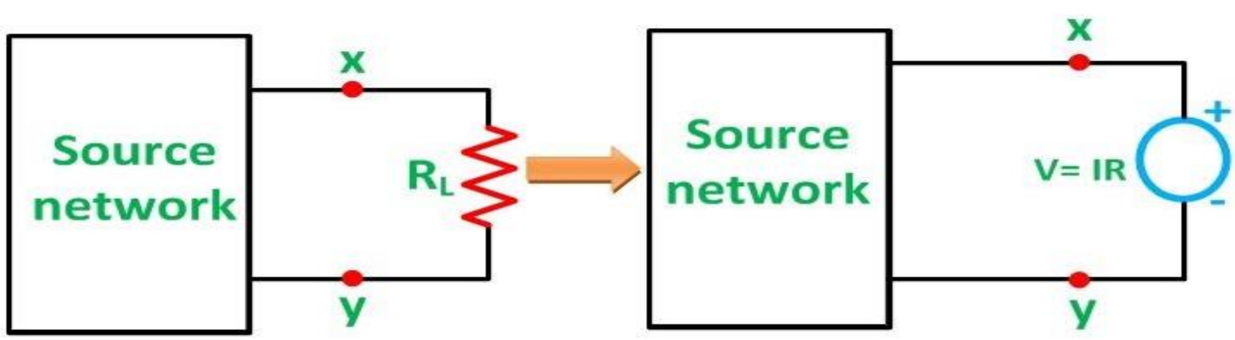
Compensation Theorem:

It is one of the important theorems in Network Analysis , which finds its application mostly in calculating the sensitivity of electrical networks & bridges and solving electrical networks. In many circuits, after the circuit is analyzed, it is realized that only a small change need to be made to a component to get a desired result. In such a case we would normally have to recalculate. The compensation theorem allows us to compensate properly for such changes without sacrificing accuracy.

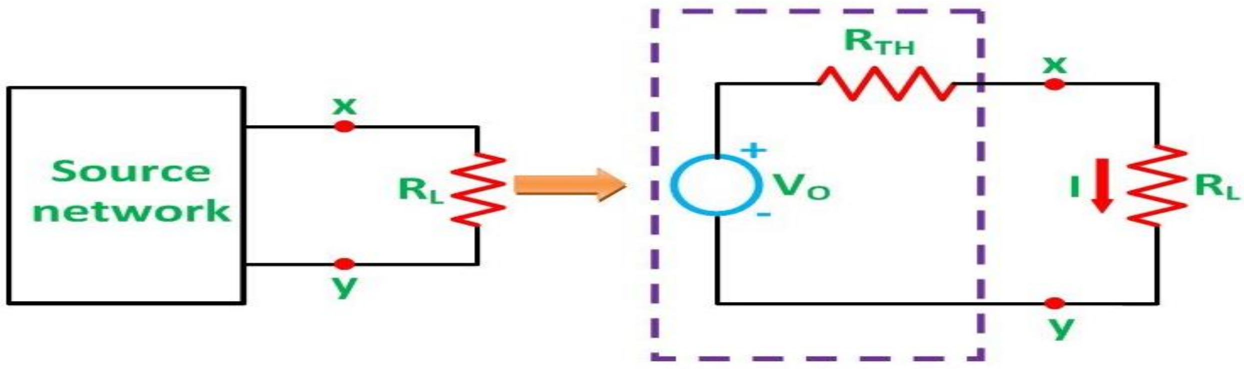
Compensation Theorem Statement:

In a linear, bilateral, time invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR). The currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (VC) has been connected such that VC = I (ΔR) in series with (R + ΔR) when all other sources in the network are replaced by their internal resistances.

In Compensation Theorem, the source voltage (V_C) opposes the original current. In simple words compensation theorem can be stated as – the resistance of any network can be replaced by a voltage source, having the same voltage as the voltage drop across the resistance which is replaced.



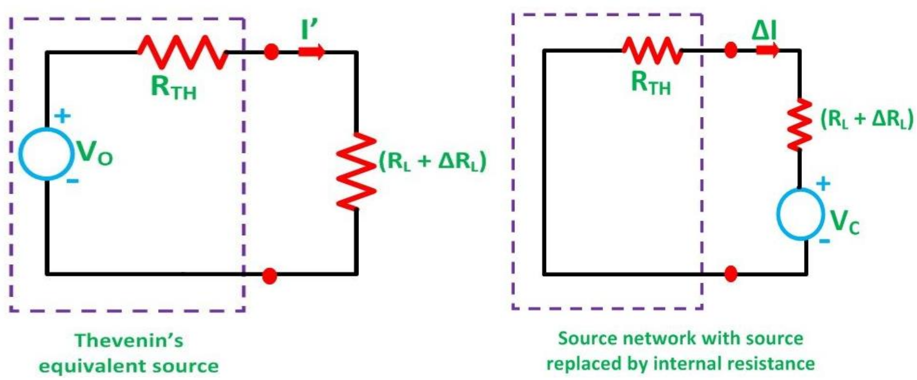
Let us assume a load R_L be connected to a DC source network whose Thevenin's equivalent gives V_0 as the Thevenin's voltage and R_{TH} as the Thevenin's resistance as shown in the figure below.



Here,

$$I = \frac{V_0}{R_{TH}+R_L} \dots\dots\dots\dots\dots..(1)$$

Let the load resistance RL be changed to (RL + ΔRL). Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the circuit diagram below



Here,

$$I' = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} \dots\dots\dots\dots\dots\dots\dots..(2)$$

The change of current being termed as ΔI Therefore,

$$\Delta I = I' - I \dots\dots\dots\dots\dots\dots\dots\dots\dots..(3)$$

Putting the value of I' and I from the equation (1) and (2) in the equation (3) we will get the following equation.

$$\Delta I = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} - \frac{V_0}{R_{TH} + R_L}$$

$$\Delta I = \frac{V_0\{(R_{TH}+R_L)-(R_{TH}+(R_L+\Delta R_L)\}}{(R_{TH}+(R_L+\Delta R_L))\times(R_{TH}+R_L)}$$

$$\Delta I = -\left[\frac{V_0}{R_{TH}+R_L}\right]\frac{R_{TH}}{R_{TH}+(R_L+\Delta R_L)}\ldots\ldots..(4)$$

Now, putting the value of I from the equation (1) in the equation (4), we will get the following equation.

$$I = -\frac{IR_{TH}}{R_{TH}+(R_L+\Delta R_L)}\ldots\ldots\ldots\ldots.(5)$$

As we know, $V_C = I\ \Delta\ RL$ and is known as compensating voltage. Therefore, the equation (5) becomes.

$$\Delta I = \frac{-V_C}{R_{TH} + (R_L + \Delta R_L)}$$

Hence, Compensation Theorem tells that with the change of branch resistance, branch currents changes and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

Example: Determine the current flowing through the ammeter having an internal resistance of 1 Ω connected in series with a 3 Ω resistor as shown in the fig (a).?

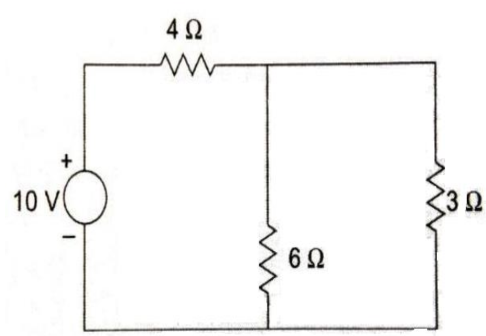
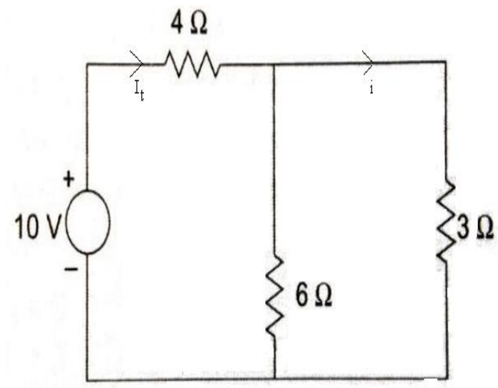


Fig.(a)

Solution:



The current flowing through the 3 Ω branch i,

$$i = I_t\,[6/(6+3)]$$

$$I_t = \frac{10}{(4+(6\|3))}$$

$$I_t = \frac{10}{(4+2)}$$

$$I_t = 1.67A$$

$$i = 1.67[6/(6+3)]$$

$$i = 1.11A$$

Now when we connect the ammeter with an internal resistance of 1 Ω in the 3 Ω branch ,there is a change in resistance . This change in resistance causes currents in other branches as if a voltage source of voltage v is

V= i. R = 1.11x1 = 1.11v

1.11v is inserted in the 3 Ω branch as shown in the fig (1) below.

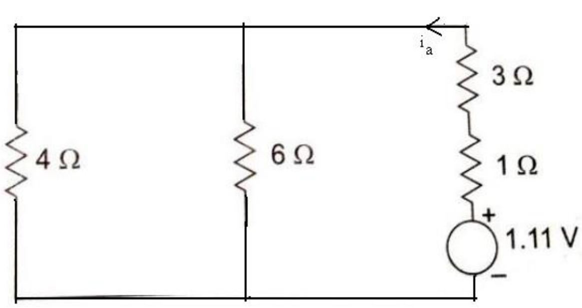


Fig.(1)

Current due to this additional source of 1.11 V in the 3 Ω branch i_a is,

$$i_a = \frac{1.11}{(1+3+(6\|4))}$$

$$i_a = \frac{1.11}{(1+3+2.4)}$$

i_a=0.17A

This current flows in the opposite direction to that of the original current i through the 3 Ω branch(i.e. i_a is opposite to i)

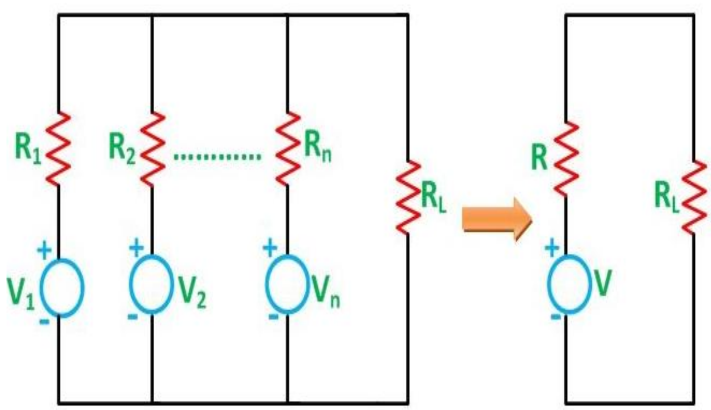
Hence Ammeter reading = i_a-i=(1.11 – 0.17) = 0.94 A

Millman's Theorem:

Millman's Theorem is a theorem which helps in simplifying electrical networks with a bunch of parallel branches. The utility of this theorem that, any number of parallel voltage sources can be reduced to one equivalent one.

Millman's Theorem Statement:

The Millman's Theorem states that – when a number of voltage sources (V_1, V_2, V_3……… V_n) are in parallel having internal resistance (R_1, R_2, R_3…………R_n) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.

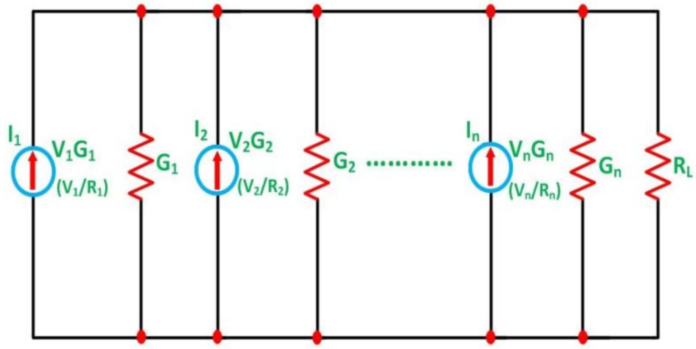


As per Millman's Theorem,

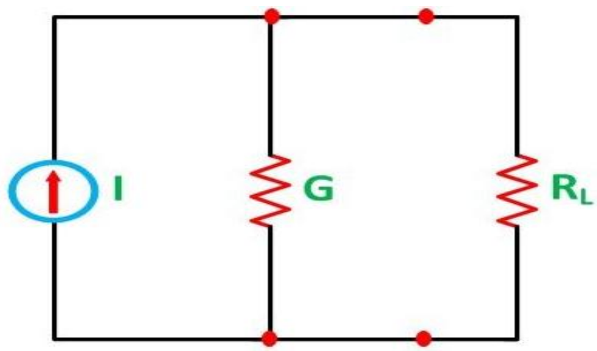
$$V = \frac{\pm V_1G_1 \pm V_2G_2 \pm \cdots \ldots \ldots \ldots \pm V_nG_n}{G_1 + G_2 + \cdots \ldots \ldots \ldots + G_n} ;$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots \ldots \ldots \ldots + G_n}$$

A DC network of numerous parallel voltage sources with internal resistances supplying power to a load resistance RL as shown in the figure below.



Let I represent the resultant current of the parallel current sources while G the equivalent conductance as shown in the figure below.



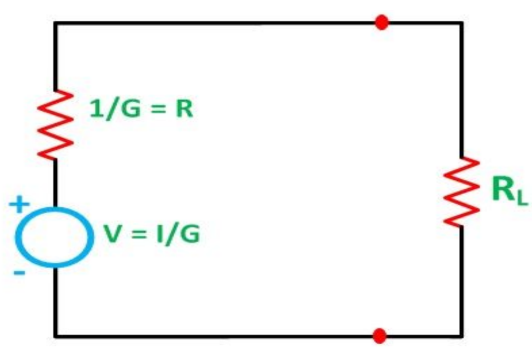
$$I = I_1 + I_2 + I_3 \ldots\ldots\ldots\ldots\ldots;$$

$$G = G_1 + G_2 + G_3 \ldots\ldots\ldots\ldots\ldots\text{(or)}$$

$$I = \frac{\pm I_1R_1 \pm I_2R_2 \pm \cdots\ldots\ldots\ldots \pm I_nR_n}{R_1 + R_2 + \cdots\ldots\ldots\ldots + R_n};$$

$$G = \frac{1}{R} = \frac{1}{R_1 + R_2 + \cdots \ldots\ldots\ldots + R_n}$$

The resulting current source is converted to an equivalent voltage source as shown in the fig.



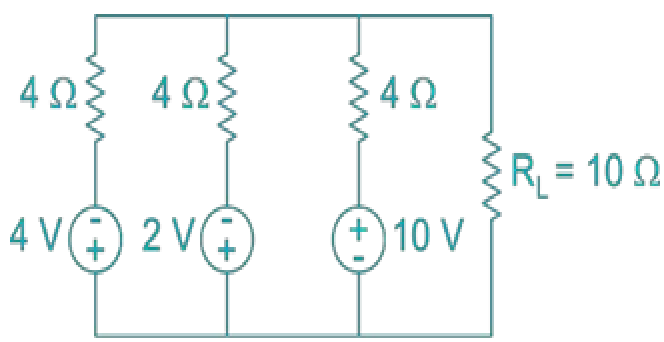
$$V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm \cdots \dots \dots \dots \pm I_n}{G_1 + G_2 + \cdots \dots \dots \dots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots \dots \dots \dots + G_n}$$

And as we know,I = V/R, and we can also write R = 1/G as G = 1/R So the equation can be written as,

$$V = \frac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \cdots \dots \dots \dots \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \dots \dots \dots + \frac{1}{R_n}}$$

Example: Find the value of current through R_L using Millman's theorem.?



Solution:

Given $R_1 = R_2 = R_3 = 4$

$G = G_1 + G_2 + G_3$

$$G = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$\therefore R = \frac{1}{G} = \frac{4}{3}\,\Omega$$

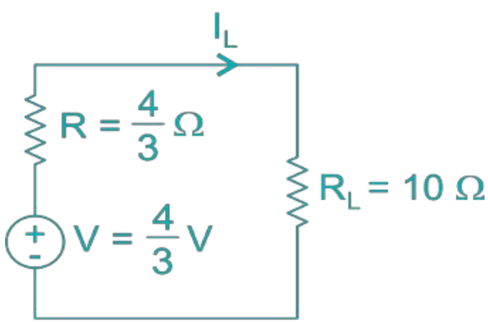
$$V = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3}$$

$$V = \frac{(-4)\frac{1}{4} + (-2)\frac{1}{4} + (10)\frac{1}{4}}{\frac{3}{4}}$$

$$V = \frac{-4 - 2 + 10}{3}$$

$$V = \frac{4}{3}$$

So given circuit becomes,



$$\therefore I_L = \frac{V}{R+R_L} = \frac{\frac{4}{3}}{\frac{4}{3}+10} = \frac{4}{34} = 117.64\text{mA}$$

The current flowing through R_L is **117.64mA**